A Model of Infrastructure Financing

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Abstract

Infrastructure projects often require large investments, can have long gestation periods, and typically involve multiple parties: government, private sector firms and outside investors. Government and private sector firms must expend effort to implement and maintain the projects, may derive private benefits in participating in the project, and may possess only limited information. Moreover, governments’ ability to finance infrastructure may be limited by fiscal deficits. These factors can potentially limit the willingness of the outside investors to supply capital. In this paper, we survey the approaches to infrastructure financing that have been developed in different parts of the world and offer a simple theory, which takes into considerations some of the ground realities in infrastructure investment projects. We show that government guarantees (for “bad states”) coupled with taxation of project revenues (in good “states”) mitigate the double moral hazard problem and leads to greater private sector investments in infrastructure projects.

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1 Introduction

1.1 Need for Infrastructure Investment

Infrastructure investments are characterized by large capital intensive natural monopolies such as highways, railways, water and sanitation systems.\(^1\) It is widely held that there are significant capacity constraints on existing infrastructure, in many developing countries. The shadow costs of such constraints on economic growth, while very hard to quantify, can be rather high. In the context of India, while some improvements have occurred over the last decade in infrastructure, the state of infrastructure facilities is generally well below what one observes in many developing economies. The inadequate state of infrastructure in India has been well documented and understood. The projected investment requirements for infrastructure are placed at $1 trillion in the 12\(^{th}\) plan and the funding gap is estimated to be above Rs. 5000 billion. It is anticipated that about half of the investment requirements of infrastructure would have to be met through funding from the private sector, and that the share of private sector in infrastructure investment will have to rise substantially from about 37 per cent in the 11\(^{th}\) Plan to about 48 per cent in the 12\(^{th}\) Plan.\(^2\) Some of the constraints that have stymied the investments in infrastructure projects are regulatory, political and legal in nature. In addition, the absence of or insufficiency of user fees is yet another constraint. The fact that infrastructure projects often require the interaction of government (in acquiring land, for example), private sector firms (in executing the construction of highways, for example), and private investors (for funding and supplying capital), makes the problem a challenging one, as we will demonstrate below. We outline in the next section the main characteristics of infrastructure projects.

1.2 Characteristics of Infrastructure Financing

Infrastructure projects typically involve very high levels of capital investments. These investments are usually sunk, and the attainment of steady-state revenues from such projects may take several years. Moreover, infrastructure investments often may require significant acquisition of land and other properties, which may be in private

\(^1\)See Gramlich (1994) for some alternative measures of infrastructure investments.

\(^2\)See, “Infrastructure Financing By Banks In India: Myths and Realities”, Keynote address delivered by Dr. K.C. Chakrabarty, Deputy Governor, Reserve Bank of India at the Annual Infrastructure Finance Conclave organized by SBI Capital markets Limited at Agra on August 9, 2013. See also Lakshmanan, L. (2008), for public-private partnerships in infrastructure investments in India.
hands. The government may be able to acquire such resources through compensa-
tions in the interest of “public good” or positive externalities that may be created through the provision of infrastructural services. In the absence of government ini-
tiatives, it is very hard to imagine why private sector will act on its own to make such investments. In turn, this can lead to market failures. Assets created through infrastructure investments are often immobile and cannot easily be transferred to other locations without incurring significant costs. For such investments as highways, bridges, tunnels and metros, the question of transferability is simply not a realistic consideration. The heavy initial investments (sunk costs) cause the average costs of infrastructure projects to differ markedly from their marginal costs in steady state. This sets in motion potentially conflicting objectives for different stakeholders. The government, which typically makes bulk of the initial investment would like to recover the sunk costs. The consumers (present and especially the future consumers) would rather pay the low marginal costs of using the facility once it is completed. Investors from private sector will be concerned about the commitment of the government to enforce tariffs and tolls from future consumers so that their costs are fully recovered and they earn a fair rate of return. This is the classic “time inconsistency” problem that the government must solve through credible commitment tools.

Over a period of time, in many infrastructure investments governments and pri-
ivate sector firms have come together in varying contractual arrangements to design and execute infrastructure projects. We will briefly review some of the contractual arrangements that have been used in other countries to provide a perspective.

In the United States the Transportation Infrastructure Finance and Innovation Act (TIFIA) of 1998 established a Federal credit program for transportation projects of national or regional significance. The idea behind this act is to attract private capital and thereby leverage the capital provided by the government at a cost that cannot be matched by the private sector acting alone. TIFIA provides three types of financing arrangements: secured direct loans to the sponsors of the project, loan guarantees to institutional investors who make loans to the project, and long-term standby lines of credit that may be drawn by the sponsors of the project. TIFIA facil-
ities have a relatively low cost, usually tied to the 10-year Treasury rates. Since 1998, TIFIA has provided over $8 billion credit for highway, transit, and other projects, mainly backed by user fees and tolls.

Many projects at State and City levels, such as highways, bridges, etc. are funded through the issuance of municipal bonds. They typically fall into two categories: the so-called general obligation (GO) bonds, which depends on the tax revenues of the
State or City for its integrity. Or, the so-called revenue bonds, which depends on user-fees such as tolls. A unique innovation is the tax treatment of these bonds: the interest income from municipal bonds are tax-exempt from the perspective of private investors, and the bonds are typically insured by mono line insurance companies. Together, these two features (and the enforcement of contractual obligations, ex-post) have allowed the development of a fairly big municipal bond market, which offers a major source of funding of infra-structure projects in the United States.

In the U.K., the Treasury has established since 2009 a unit that co-lends along with private sector lenders to fund privately financed initiatives (PFI). This is a model of government co-investing with the private sector to help the infrastructure projects to accomplish a closure of their initial financing. The stated goal is to be able to exit the investment by selling the loans in the private capital markets once the projects become self-sustaining.

In France, a two-pronged approach is used to finance infrastructure projects through public sector, private sector partnership (PPP) programs. First, the French government has provided a 8 billion Euro guarantees to bank loans that are directed towards infrastructural projects. This allows commercial banks to provide funding to private sector sponsors of infrastructure projects. Second, the government has established a 10 billion Euro guarantees to promote debt financing. These guarantees perform two functions. First, they promote the liquidity of the market for bank loans and bonds. Second, through government guarantees, infrastructure projects can be funded at relatively low costs.³

Similar contractual arrangements are used in Australia in their PPP programs to fund infrastructure projects. Australia has co-lending facilities, whereby it lends on commercial terms (along with private sector banks) to fill the funding gap. In this arrangement, the government plans to exit over a period of 3 to 5 years. Australia also has a guarantee program to address the funding gap in infrastructure financing. In both co-lending and in the guarantee program, the government recognizes the liability created by these financing arrangements. In addition, outright cash subsidies are also provided for some infrastructure projects.⁴

⁴ See “Infrastructure Partnerships Australia: Financing Infrastructure in the Global Financial Crisis,” (2009), March.
Infrastructure financing has been a topic extensively studied by policy makers and practitioners. A number of papers have addressed how large-scale infrastructure projects can be organized and financed.

The theoretical literature on infrastructure financing has explored the question of whether the project should be exclusively organized by the government or be structured in partnership with players from the private sector. Martmort and Sand-Zantman (2006) consider the classic infrastructure problem in which the government is seeking to procure a public good or service on behalf of its citizens. The government may either deliver the service under public ownership (railways, water, or power, for example) or fully or partially outsource the activity to the private sector. Martmort and Sand-Zantman (2006) examine the contractual forms that such delegated management may take when the quality of the infrastructure is key to the social value of the service. Their model has the following trade-off: retaining good projects is a way for the government to signal to private parties that the quality of the infrastructure assets is good. But this comes at the expense of moral hazard entailed by imperfect information and non-verifiability of efforts. Their model delivers the following sharp predictions: first, the amount of risk kept by the government increases with the quality of the infrastructure. Full privatization emerges in their model only for the worst-quality infrastructures. In their analysis, the government does not face any explicit financing constraints, which is a matter of some importance for our paper.\(^5\)

Perotti (1995) provides a framework in which partial privatization is a way for a government to credibly signal that it will not behave opportunistically upon privatization (such as decreasing or even eliminating tolls, once the toll-highways are privatized). The profits are assumed to be exogenous in this model. In reality, profits may be endogenous depending on the efforts expended by both parities - the private sector player must keep the highways in good order, and the government must commit not to behave in a politically opportune manner, ex-post.\(^6\)

We can think of some infrastructure projects as jointly owned investment options. The government may own vast tracts of land, and private sector firms may wish


\(^6\)For example, after awarding the private sector firms highway contracts, government may, under political pressure, reduce the tolls to unacceptably low levels, placing the survival of private sector firms at risk. Infrastructure projects with user fees are subject to this risk.
to develop it for commercial purposes. In such situations, the investment timing decisions will interact with the bargaining power of the parties involved, any side payments that the parties may agree to make to each other, and the manner in which the net present value of the projects will be divided up. Banerjee, Gucbilmez, Pawlina (2012) provide a real-options framework to investigate the optimal investment timing in the presence of joint ownership, bargaining and side payments. Medda (2007) argues that in the case of large-scale public-private partnerships, if the guarantees provided exceed the potential financial losses of private sector, it can lead to strategic behavior and lead to problems of moral hazard.

3 Modeling Issues

The following points emerge from the literature from a modeling perspective. First, there are financing and other constraints (such as expertise) that preclude the government from pursuing infrastructure investments on its own. This implies that private sector will be a key player both in financing the infrastructure projects and in executing and maintaining infrastructure projects. Second, the potential revenue generated from the projects depend on the efforts expended by both private sector players vested with the task of maintaining and upgrading the infrastructure facility as well as the government’s commitment to not to make unilateral decisions to reduce or eliminate user fees. The private sector investors will clearly take these issues into account in deciding whether to finance projects at an attractive rate or not. Third, given the scale of investment that is required, and the possibility that the government may run a fiscal deficit, it may be critical for the government to offer some incentives (such as tax exemption of infrastructure bonds, financial guarantees for a specified period of time, etc.) to the private sector entities involved in the execution and investment process.

We incorporate the following key and distinctive features of infrastructure investments. First, we explicitly allow for the possibility that the government must expend some effort in the eventual success of infrastructure investments. The government may derive some private benefits from the projects’ investment process. We also explicitly model the government to be financially constrained. Second, we also model that the private sector firms, which implement the project also will have to expend efforts and they may also reap some private benefits. In this sense, there is a “double moral hazard” problem that is present as recognized in the literature: the government and the private sector do not have the same information set and they must
expend effort. In addition, in the context of infrastructure investments, there is a political dimension as well, which may encourage the government to behave in an opportunistic fashion: for example, after a project is implemented, (say, a highway), the government may choose to give “toll holidays” to appease voting public in an effort to win elections. This may require that the contract is written in such a way that it is “iron clad” vis-a-vis such possibilities. One mechanism might be to set aside autonomous special purpose vehicle (SPV), with guarantees of user fees, backed by government collateral. On the other hand, one must simultaneously ensure that the private sector firms, which maintain the project do so with diligence, making sure that the guarantee funds are only available to them upon meeting some verifiable markers attesting to the proper upkeep and maintenance of the facilities. The issue of verifiability can be non-trivial. For projects in which water or power is supplied, it may be a lot easier to verify the amount and quality of delivery. For highways and bridges, it may be more costly to put in place a verification mechanism in place.

4 Model Ia

We first develop a benchmark model providing the rationale for why government guarantees would be essential for efficient financing of infrastructure finance.

4.1 Setup

Consider a two-stage infrastructure project. The project is run by a private project operator (which we will refer to simply as the “private sector”). However, in the first stage, the project requires government “input.” This input can represent project approval, land acquisition, clearance of existing properties on the land, provision of public utilities, etc. In the second stage, that is, once the project has gone past the government input stage, the private sector can shape the quality of the project based on its own inputs. Hence, there is the potential for double moral hazard. In particular, both government-sector and private-sector inputs will be provided at efficient levels only if each has incentives to do so. More interestingly, the two inputs will interact in determining the project payoffs, and, in turn, affect both sectors’ incentives.

However, with some exceptions, user fees are invariably subsidized at levels well below marginal cost. Alm (2010) notes that the problems that lead to this outcome, include inadequate billing and collection procedures, insufficient attention to operations and maintenance, and political constraints.
The project is funded by private investors who rationally anticipate that the returns they will receive are affected in expectation by the strength of the government-sector and private-sector incentives. The private investors also take into account any provision of government guarantees to their financing of the infrastructure project. Such guarantees will expose the government to the risk of project failure and potentially ameliorate the government moral hazard. However, the size of the guarantees is limited by the fiscal constraint that the government faces in making available its balance-sheet for infrastructure finance.\(^8\)

Formally, the project is constant returns to scale. Denote the scale of the project as \(I\). In the first stage, the government through its input can affect the probability \(e\) of the project’s success. If the government input is high, the project succeeds with a probability \(e_h \in (0, 1)\), else with a probability \(e_l, 0 < e_l < e_h\). We will denote as \(\Delta e\), the difference in these probabilities: \(\Delta e \equiv (e_h - e_l)\). If the government does not exert the high input, the associated officials are assumed to derive a non-pecuniary private benefit of \(bI, b > 0\). In case the project fails in the first stage, it has no further chance of success and its payoff is zero.

Providing that the project has not failed in the first stage, the private sector can affect the probability \(p\) of the project’s eventual success. If the private sector input is high, the project’s conditional probability of success in the second stage is \(p_h \in (0, 1)\), else it is \(p_l, 0 < p_l < p_h\). We will denote as \(\Delta p\), the difference in these conditional probabilities: \(\Delta p \equiv (p_h - p_l)\). If the private sector does not exert the high input, it derives a non-pecuniary private benefit of \(BI, B > 0\). In case the project fails in the second stage, its payoff is zero. And in case it succeeds eventually, its payoff is \(RI > 0\).

Since there are no cash flows after the first stage of the project, the only way the government can ameliorate its moral hazard problem is by either incurring a cost in case of project’s failure after the first stage or having an incentive payoff if the project succeeds after the second stage. To model the first possibility, we assume that the government can provide a guarantee to the private investors of \(KgI\) in case the project fails in the first stage. We assume that the size of this guarantee is constrained by the fiscal capacity of the government that in no state can this liability exceed an (un-modeled) upper limit \(\bar{K}\). To model the second possibility, we assume that the

\(^8\)In addition, project’s success could also depend on other factors such as unexpected delays in court decisions about the legality of acquisition of lands for highways, or autonomous changes in prices of inputs. If the scale of the project is too high, multi-lateral guarantees may be the only feasible solution.
government can obtain a share of the project’s payoff in case of success, which can be interpreted as taxes for instance. We denote the government’s payoff as $R_g I$, where $R_g \leq R$.

The private investors thus receive a government guarantee in case the project fails after the first stage. They are also offered a return $R_b I$ in case the project succeeds eventually, such that $(R_b + R_g) \leq R$. The residual payoff in case of project’s success, $(R - R_b - R_g) I$, accrues to the private sector project operator and will serve to incentivize them to exert effort in the second stage of the project. Finally, we assume that both the private investors and the government require a net rate of return on their respective investments (in case of government, the contingent investment in the form of the guarantee) in the project. To start with we assume these rates of return to be identical for the private investors and the government and normalize it to zero.\(^9\)

The state space of outcomes for the projects, and project payoffs as well as payoffs to various parties (the private sector project operator, the private investors, and the government) are summarized in Figure 1.

\(^9\)It is possible that the government may take into account the positive externalities created by infrastructure projects (in creating employment, or improving the livelihood of citizens living where the facilities are built). It is therefore likely that their discount rate could be lower than that of the agents in the private sector. We allow this possibility later in the paper.
4.2 Analysis

To analyze the properties of the model, we consider in turn the two incentive constraints, the two individual rationality constraints, and the government’s fiscal constraint.

1. The private sector’s incentive constraint in the second period is that after the first stage of the project, its expected returns from exerting the high effort must not be dominated by its expected returns (inclusive of the private benefits) from exerting the low effort:\(^\text{10}\)

\[
p_h (R - R_b - R_g) I \geq p_l (R - R_b - R_g) I + BI,
\]

\[(\text{IC–pvt})\]

\(^\text{10}\)We take the view that the private sector owns and operates the infrastructure facility, if and when it gets on stream. This explains why the private sector gets the residual returns.
or
\[ R - (R_b + R_g) \geq \frac{B}{\Delta p}. \]  \hspace{1cm} (1)

In other words, enough project cash flows upon success must be left as residual claim for the private sector for it to have incentives to ensure the project operates at a high probability of success in the second period.

2. The government’s incentive constraint in the first period is that its (second-period) expected returns from exerting the high effort in the first stage must not be dominated by its expected returns (inclusive of the private benefits and net of the cost of providing guarantees to the investors) from exerting the low effort:

\[ e_h p_h R_g I + (1 - e_h) K_g I \geq e_l p_h R_g I + (1 - e_l) K_g I + bI, \]  \hspace{1cm} (IC–govt)

or

\[ p_h R_g + K_g \geq \frac{b}{\Delta e}. \]  \hspace{1cm} (2)

That is, the government’s conditional expected share of the project payoff in the second period and the penalty it suffers from providing the guarantee to investors must be sufficiently high to counteract its moral hazard in providing inputs to the first stage of the project.

3. The private investors, however, must also be left with adequate share of the project payoff, so that this expected share plus the expected value of the government guarantee compensate the investors for an adequate rate of return on their investment in the project. This yields the private investors’ individual rationality constraint:

\[ I \leq e_h p_h R_b I + (1 - e_h) K_g I, \]  \hspace{1cm} (IR–inv)

or

\[ e_h p_h R_b + (1 - e_h) K_g \geq I. \]  \hspace{1cm} (3)

4. The government’s individual rationality constraint must also be satisfied so that it is not losing money relative to its required rate of return, net of taxes and the guarantees:

\[ [e_h p_h R_g - (1 - e_h) K_g] I \geq 0, \]  \hspace{1cm} (IR–govt)
or
\[ K_g \leq \frac{e_h p_h R_g}{1 - e_h}. \]  
(4)

In other words, taxes cannot be too low relative to the guarantee the government has provided the private investors.

5. Finally, the guarantee that the government provides cannot exceed its fiscal constraint when the guarantee has to be honored:11
\[ K_g I \leq K, \]  
(Fiscal-constraint)

or
\[ I \leq \frac{K}{K_g}. \]  
(5)

As is clear, the fiscal constraint limits the scale of the investment for a given size of the (per-unit) guarantee provided by the government to the private investors. This is natural as absent the fiscal constraint, the government can always ameliorate its moral hazard problem by setting the guarantee to be sufficiently high and any project scale can then be supported. But this is unrealistic in the scenario where the guarantees must be honored in case of project failure.

Thus, the contracting problem for infrastructure finance in our model has four degrees of freedom:
\[ R_b, \quad R_g, \quad K_g \geq \phi, \quad I. \]

From societal standpoint, the objective of the contracting problem is to maximize the net present value of the infrastructure project, that is, its expected payoff net of investment (as all other payoffs are simply transfers between the government and the private sector):
\[ \max [e_h p_h R I + e_h (1 - p_h) .0 - I] \]  
(6)

\[ = \max (e_h p_h R - 1) I \]  
(7)

11Government’s fiscal constraint places an aggregate limit on how much it can allocate to infrastructure sector as a whole. This in turn, will lead to some limits on individual projects. We treat the constraint at an individual project level, and abstract from broader issues as to which projects get funded or not.
subject to the five constraints enumerated above. Four of the five constraints (excluding say, the government’s individual rationality constraint) can be solved to yield a closed-form solution:

**Lemma 4.1** The solution to the contracting problem above is given by,

\[
K_g^* = \left[1 - e_h p_h \left( R - \frac{B}{\Delta p} - \frac{b}{p_h \Delta e} \right) \right]; 
\]

\[
R_g^* = \left[ \frac{b}{p_h \Delta e} - \frac{K_g^*}{p_h} \right]; 
\]

\[
R_b^* = \left[ \left( R - \frac{B}{\Delta p} \right) - R_g^* \right]; \text{ and,} 
\]

\[
I^* = \frac{K}{K_g^*}. 
\]

Substituting these in (IR–govt) implies that the government’s individual rationality constraint is satisfied if and only if

\[
e_h p_h \left( R - \frac{B}{\Delta p} \right) \geq 1, 
\]

which is the condition that the project will be funded by investors in the absence of any government moral hazard in the first stage.

Finally, we can identify conditions under which \( K_g^* > 0 \), that is, the government moral hazard is severe enough to require some government guarantee in the optimal contracting outcome. This is equivalent to requiring that

\[
1 > e_h p_h \left( R - \frac{B}{\Delta p} - \frac{b}{p_h \Delta e} \right), \text{ or} 
\]

\[
\left( R - \frac{B}{\Delta p} \right) < \frac{b}{p_h \Delta e} + \frac{1}{e_h p_h}. 
\]

We then obtain the following characterization (shown also in Figure 2):

**Proposition 4.2** The feasibility of infrastructure finance for the project depends upon the project return \( R \) as follows:
1. If \( R - \frac{B}{\Delta p} < \frac{1}{e_h p_h} \), then the project is not funded by private investors even in the absence of any government moral hazard in the first stage of the project.

2. If \( R - \frac{B}{\Delta p} \in \left[ \frac{1}{e_h p_h}, \frac{b}{p_h \Delta e} + \frac{1}{e_h p_h} \right) \), then the project is funded by private investors and the government needs to provide guarantee to investors: \( K_g^* > 0 \). The scale of the project, however, is limited to \( I^* = \frac{K}{K_g} \).

3. If \( R - \frac{B}{\Delta p} \geq \frac{b}{p_h \Delta e} + \frac{1}{e_h p_h} \), then the project is funded by private investors without the need for any government guarantee: \( K_g^* = 0 \). In this case, there is no limit due government moral hazard and fiscal constraint on the scale of the project.

Figure 2: Project viability under Model I

Project not funded by private investors even absent government moral hazard

Project funded by private investors only with government guarantee to counter govt. moral hazard; Project scale limited by government’s Fiscal constraint in providing guarantee

Project funded by private investors; No government guarantee needed to eliminate the govt. moral hazard; Investment scale up to maximal scale can be attained
4.3 Discussion

The characterization of contracting outcome and feasible projects contain important effects arising from the double moral hazard nature of the problem.

Note that the government proceeds \( R_g \) and the government guarantee to private investors \( K_g \) both ameliorate the government moral hazard problem. The proceeds serve as an incentive or the “carrot” for the government to facilitate the project in the first stage, whereas the guarantee serves as a disciplining device or the “stick” for the government to avoid project failure. The reason why the incentive mechanism does not suffice and government guarantee is needed is entirely due to the moral hazard faced by the private sector operator in ensuring high project quality in the second stage. Absent this second moral hazard, the government would not need to leave any incentive share from payoff for the private sector operator and instead collect the payoff itself, which in general could induce efficient effort from the government in the first stage.

The greater the private sector moral hazard, the greater the share of proceeds that has to be left as an incentive for the private sector. This constrains the contracting outcome from relying exclusively on \( R_g \) and necessitates a role for the government guarantee in the form of a positive guarantee: \( K_g > 0 \). In the same vein, the greater the private sector moral hazard, the less the proceeds are available to provide the required rate of return through \( R_b \) to private investors. In turn, the private investors must also be provided their required rate of return through a greater extension of the government guarantee. However, since the government guarantee is constrained by the fiscal constraint, the greater the per-unit guarantee required in the contract, the smaller is the feasible scale of the project.

Finally, the greater the government moral hazard, the greater is the guarantee that it must provide, as all else equal the government cannot extend to itself a greater share of the proceeds beyond a point without violating the private sector’s incentive constraint (IC–pvt) or the private investors’ individual rationality constraint (IR–inv).

These intuitions stemming from the double moral hazard nature of our setup for infrastructure finance are summarized below (where we have underlined the effects that are distinct between the private sector and the government moral hazard).

**Corollary 4.3** As the private sector moral hazard \( (\frac{B}{\Delta p}) \) increases,

- The required government guarantee \( K_g^* \) increases;
- The government proceeds from the project payoff \( R_g^* \) **decrease**;
• The private investors’ proceeds from the project payoff $R_b^*$ decrease; and,
• The scale of investment $I^*$ decreases.

**Corollary 4.4** As the government moral hazard ($\frac{b}{\Delta e}$) increases,

• The required government guarantee $K_g^*$ increases;
• The government proceeds from the project payoff $R_g^*$ increase;
• The private investors’ proceeds from the project payoff $R_b^*$ decrease; and,
• The scale of investment $I^*$ decreases.

### 5 Model Ib: Differential rate of return for private investors and government

Suppose the private investors have a greater required rate of return $r > 1$, compared to that of the government. This can capture externalities from infrastructure projects not internalized by the private investors, as well as other frictions requiring higher rate of return due to opportunity costs faced by private investors. For instance, if the private investors are banks, this could reflect the capital requirements and liquidity surcharges imposed on banks for making loans, which can extend to infrastructure loans. These, in turn, can induce a greater required rate of return by banks compared to the government’s required rate of return from the infrastructure project.

We show below the intuitive result that such fraction raises the size of the government guarantee, and thereby reduces the scale of investment and the set of viable projects compared to the case with $r = 1$.

The constraint that changes is the individual rationality constraint of the private investors (IR–inv) which takes the form:

$$rI \leq e_h p_h R_b I + (1 - e_h) K_g I \quad \text{(IR–inv–2)}$$

$$\iff e_h p_h R_b + (1 - e_h) K_g \geq r. \quad \text{(15)}$$

All other constraints remain unaffected. It is then straightforward to show the following set of intuitive results.
Lemma 5.1 The solution to the contracting problem when the private investors require a rate of return \( r \) is given by

\[
K_g^*(r) = \left[ r - e_h p_h \left( R - \frac{B}{\Delta p} - \frac{b}{p_h \Delta e} \right) \right],
\]

where \( R_g^* \), \( R_b^* \) and \( I^* \) are given in terms of \( K_g^*(r) \) as in (4.1).

Corollary 5.2 As the private investors’ required rate of return \( r \) increases,

- The required government guarantee \( K_g^* \) increases;
- The government proceeds from the project payoff \( R_g^* \) decrease;
- The private investors’ proceeds from the project payoff \( R_b^* \) increase; and,
- The scale of investment \( I^* \) decreases.

Importantly, the project viability is affected too. In particular, it can be shown that the (IR–govt) is met only if

\[
e_h p_h \left( R - \frac{B}{\Delta p} \right) \geq r > 1,
\]

so that projects with payoff \( R \) such that \( \left( R - \frac{B}{\Delta p} \right) \in \left[ \frac{1}{e_h p_h}, \frac{r}{e_h p_h} \right] \) do not simultaneously satisfy (IR–inv–2) and (IR–govt), and are therefore unviable.

Furthermore, the government guarantee is needed for some positive net present value projects that with \( r = 1 \) could be funded without a government guarantee. The full characterization is as follows (shown also in Figure 3):

Proposition 5.3 The feasibility of infrastructure finance for the project depends upon the project return \( R \) and private investors’ required rate of return \( r \) as follows:

1. If \( \left( R - \frac{B}{\Delta p} \right) < \frac{r}{e_h p_h} \), then the project is not funded by private investors even in the absence of any government moral hazard in the first stage of the project;

2. If \( \left( R - \frac{B}{\Delta p} \right) \in \left[ \frac{r}{e_h p_h}, \frac{b}{p_h \Delta e} + \frac{r}{e_h p_h} \right) \), then the project is funded by private investors and the government needs to provide guarantee to investors: \( K_g^*(r) > 0 \). The scale of the project, however, is limited to \( I^* = \frac{K}{K_g^*(r)} \).
3. If \( \left( R - \frac{B}{\Delta p} \right) \geq \frac{b}{p_h \Delta e} + \frac{r}{e_h p_h} \), then the project is funded by private investors without the need for any government guarantee: \( K^*_g(r) = 0 \). In this case, there is no limit due government moral hazard and fiscal constraint on the scale of the project.

Figure 3: Project viability under Model Ia (with required rate of return for investors = \( r \))

6 Model II: Government moral hazard in second stage

Now, let us add government moral hazard in continuation or the second stage. Once the first stage of the infrastructure project is complete wherein the government input is crucial to ensure the sound prospects of the project in future, the government cannot commit not to “extort” in the second stage on project cash flows unless it
has incentives not to engage in such extortion. Such extortion may take the form of coercive diversion of project cash flows, retroactive taxes, restrictions on price-setting, etc., which have direct or indirect benefits to the government at the expense of cash flows left behind for the private sector operator and investors. One, such extortion will destroy continuation incentives of private management of the project. Second, it will reduce the anticipated payoff to private financiers who would therefore require a higher return in some other form.

Hence, while the possibility of such extortive behavior remains ex post, the government can commit ex ante not to engage in such behavior by committing to incur penalties in case of eventual project failure (whose likelihood is greater in case of poor maintenance by the project operator). Such penalties can take the form of government guarantees in the second stage to private investors as such guarantees ameliorate the moral hazard and also compensate the private investors for anticipated loss of return in case government engages in extortive behavior.

### 6.1 Setup

Formally, we assume that after the first stage of the project is over, the government can potentially extort and reduce the second stage cash flows available for payments to investors and the private sector operator. We denote this second-stage government guarantee per unit scale of investment as $K^p_g$. Furthermore, to distinguish it from the first-stage guarantee, we relabel the first-stage guarantee as $K^e_g$. The revised state-space of the model and project cash flows for various parties are shown in Figure 4.
6.2 Analysis

To analyze the properties of the model, we consider first the second-stage incentive constraint faced by the government as well as the second-stage fiscal constraint, and then revisit the other constraints from Model Ib.

- The government’s second-stage incentive constraint can be formalized as follows. If the government extorts in the second stage, it can still ensure the private sector implements the high probability $p_h$, providing that $R_g \leq \overline{R}_g \equiv \left( R - \frac{B}{\Delta p} \right)$. This upper bound on $R_g$ leaves sufficient cash flow for the private sector operator to be incentivized to exert effort. Hence, ex post the government will always extort up to this upper bound. However, in this case there is no residual cash flow left to pay off investors, i.e., $R_b = 0$. Note also that if the government extorts beyond this upper bound, then the private sector operator will not...
exert effort and implement $p_l$. Then, the government might as well extort the entire cash flow up to $R$.

Thus to implement $p_h$ by the private sector operator, the government needs to satisfy an incentive constraint at the beginning of the second period that ensures that it will not extort beyond the upper bound $\overline{R}_g$:

$$p_h\overline{R}_g - (1 - p_h)K^p_g \geq p_l R - (1 - p_l)K^p_g,$$  \hspace{1cm} (IC–govt–2)

or

$$K^p_g \geq \frac{(p_l R - p_h \overline{R}_g)}{\Delta p},$$

which after substituting for $\overline{R}_g$ can be further simplified to

$$K^p_g \geq \frac{p_h B}{(\Delta p)^2} - R.$$

- The government faces the fiscal constraint also in the second stage:

$$K^p_g I \leq \overline{K}.$$  \hspace{1cm} (FC–2)

Finally, we have the modified versions of various first-stage constraints:

- The first-stage incentive constraint of the government now interacts with the second-stage constraint:

$$e_h p_h \overline{R}_g - (1 - e_h)K^e_g - e_h (1 - p_h)K^p_g \geq e_L p_h \overline{R}_g - (1 - e_L)K^e_g - e_L (1 - p_h).$$  \hspace{1cm} (18)

$$\Delta e p_h \overline{R}_g + \Delta e K^e_g - \Delta e (1 - p_h)K^p_g \geq b, \text{ or}$$  \hspace{1cm} (19)

$$p_h \overline{R}_g + K^e_g \geq \frac{b}{\Delta e} + (1 - p_h)K^p_g.$$  \hspace{1cm} (IC–govt–II)

Effectively, the guarantee required to address the continuation moral hazard of the government dilutes its ex-ante incentives to ensure the first-stage project success. Hence, $K^e_g$ may have to be adjusted upward to account for the dilution from $K^p_g$. The intuition is that the government incurs the second-stage guarantee
cost when project fails eventually even if the government exerts effort in the first stage. Counteracting this force to raise $K_e$ is the fact that by extorting up to $\overline{R}_g$ in the second stage and improving its share from the project payoff, the government has greater incentive ex ante to exert effort in the first stage.

- The government must earn its required rate of return taking account of guarantees in both stages of the project and the extortion in the second stage:

\[
[e_h p_h \overline{R}_g - (1 - e_h)K_e - e_h(1 - p_h)K_g] \geq 0.
\]

(IR–govt–II)

- The government faces the fiscal constraint in the first stage of the project:

\[
K_e I \leq \overline{K}.
\]

(FC–II)

- Finally, the private investors effectively expect to be paid back only through the government guarantees as they anticipate the government extortion in the second stage:

\[
(1 - e_h)K_e + e_h(1 - p_h)K_g \geq r.
\]

(IR–pvt–II)

Fiscal constraints in the two stages can be combined to get the parsimoniously represented constraint:

\[
\max [K_e, K_g] I \leq \overline{K}.
\]

(FC–unified)

Then the scale of investment is given by

\[
I = \frac{\overline{K}}{\max [K_e, K_g]}.
\]

(20)

Therefore, we have three contracting variables, $K_e$, $K_g$ and $I$, to maximize net proceeds from investment

\[
\max [e_h p_h \overline{R} - 1] I
\]

(21)
subject to the constraints (IC–govt–2), (IC–govt–II), (IR–govt–II), (IR–pvt–II), and (FC–unified).

Note that (IC–govt–2) gives the minimum value for the second-stage guarantee directly:

$$K_p^* = \frac{p_h B}{(\Delta p)^2} - R.$$  \hfill (22)

Substituting in (IC–govt–II) and (IR–pvt–II) gives the minimum value for the first-stage guarantee:

$$K_e^* = \max \left[ \frac{b}{\Delta e} + (1 - p_h)K_p^* - p_h R_g, \frac{r - e_h(1 - p_h)K_p^*}{(1 - e_h)} \right].$$  \hfill (23)

If (IR–govt–II) is satisfied at these minimum values, then the project is feasible to finance at a scale

$$I^* = \frac{\bar{K}}{\max [K_e^*, K_p^*]}. \hfill (24)$$

**Solution to Model II**

We have not yet fully characterized the solution to this model, but here is a sketch of the results obtained so far.

The constraints (IC–govt–II) and (IR–pvt–II) both yield lower bounds on the size of the first-stage guarantee $K_e^*$. Depending on the constraint that binds, we can determine $K_e^*$. We can then verify if $K_e^*$ together with the second-stage guarantee $K_p^*$ can ensure the government earns its required rate of return so that the project is viable.

There are thus two cases to consider:

**Case 1:** When the project payoff is sufficiently high, the government incentives are provided well enough through its share of projects that the binding constraint for the first-stage guarantee is that private investors earn adequate return. This case arises when

$$\left( R - \frac{B}{\Delta p} \right) \geq \frac{b}{\Delta e} + \frac{[(1 - p_h) + \Delta p(1 - e_h)] p_h B}{(1 - e_h)(\Delta p)^2} - \frac{r}{(1 - e_h)}. \hfill (25)$$

The structure of the solution is similar to Model Ib. In particular, we obtain that
• The project is viable if and only if
\[
\left( R - \frac{B}{\Delta p} \right) > \frac{r}{e_h p_h},
\] (26)
that is, whenever it is viable in absence of the government moral hazards.

• The second-stage guarantee is required, i.e., \( K^p_g > 0 \), whenever project returns are low enough that they do not provide adequate incentives to the government in the second stage not to extort. The precise condition is
\[
\left( R - \frac{B}{\Delta p} \right) < \frac{p_l B}{(\Delta p)^2}.
\] (27)

• Next, the first-stage guarantee is greater than the second-stage guarantee, i.e., \( K^e_g > K^p_g \) if and only if the project payoff is adequately high:
\[
\left( R - \frac{B}{\Delta p} \right) > \frac{p_l B}{(\Delta p)^2} - \frac{r}{(1 - e_h)}. \tag{28}
\]
In this case, the project requires a “permanent” guarantee, \( K^p_g \), throughout the term of the project, and an additional “temporary” guarantee, \( (K^e_g - K^p_g) \), during the first stage of the project.

• Finally, when \( K^p_g > K^e_g \), the scale of the investment is constrained by the second-stage guarantee to \( I = \frac{K}{K^p_g} \).

Case 2: When the project payoff is not sufficiently high, the binding constraint for the first-stage guarantee is that the government has adequate incentives in the first stage to exert effort. This case arises when
\[
\left( R - \frac{B}{\Delta p} \right) < \frac{b}{\Delta e} + \frac{[(1 - p_h) + \Delta p(1 - e_h)] p_h B}{(1 - e_h) (\Delta p)^2} - \frac{r}{(1 - e_h)}. \tag{29}
\]
The structure of the solution is now different from Model Ib which did not feature the continuation or the second-stage moral hazard. Now, the first- and second-stage guarantees are determined by the two incentive constraints for the government, rather than being determined also by the individual rationality of private investors. In this case, we obtain that
• The project is viable if and only if

\[
\left( R - \frac{B}{\Delta p} \right) > \frac{(1 - e_h) b}{\Delta e} - \frac{(1 - p_h) p_l B}{(\Delta p)^2},
\]

that is, in general not all projects that are viable under Model Ib are viable under Model II, Case 2. The intuition is that the two government incentive constraints require such a high level of guarantees that the government would not earn the required rate of return on these investments.

• The condition that the second-stage guarantee is required, i.e., \( K_g^p > 0 \), remains the same as in Case 1.

• Next, the first-stage guarantee is greater than the second-stage guarantee, i.e., \( K_g^p > K_g^e \) if and only if the government moral hazard in the first stage is sufficiently worse than the private-sector moral hazard in the second stage:

\[
\frac{b}{\Delta e} > \frac{p_h p_l B}{(\Delta p)^2}.
\]

The intuition comes from the double moral hazard setting that when the private-sector moral hazard in the second stage is not too binding, adequate project payoffs can be left for the government to address the second-stage government moral hazard without a need for much second-stage guarantee. As in Case 1, when \( K_g^e > K_g^p \), the project requires a “permanent” guarantee, \( K_g^p \), throughout the term of the project, and an additional “temporary” guarantee, \( (K_g^e - K_g^p) \), during the first stage of the project.

• And, when \( K_g^p > K_g^e \), the scale of the investment is constrained by the second-stage guarantee to

\[
I = \frac{K_g^e}{K_g^p},
\]

else it is given by

\[
I = \frac{K_g^e}{K_g^p}.
\]
7 References


